Chapter 1: Graphs, Functions, and Models

Topic 1: Basics of Functions – p. 162

- Homework: p. 176 #1-38 Even

Topic 2: Restricted Domains – NOT IN BOOK

- Homework: Worksheet

Topic 3: Difference Quotient – p. 193

- Homework: p. 198 #71-86

Topic 4: Identifying Key Features of a Graph – p. 181

- Homework: p. 195 & 196 #1-48 Even

Topic 5: Shifts in Graphs – p. 229

- Homework: p. 241 &242 #1-44 Even

Topic 6: Combinations & Composite Functions – p. 246

- Homework: p. 258 #51-71 Even

Topic 7: Inverse Functions – p. 261

- Homework: p. 269 & 270 #1-50 Even

Topic 8: Circles – p. 273

- Homework: p. 280 #1-52 Even

Name: ______________________________ Date: ______________ Period: ______

Chapter 1: Graphs, Functions & Models
Topic 1: Basics of Functions

Relations & Functions

A **relation** is a relationship between sets of information. It is a set of ordered pairs.
Recall:

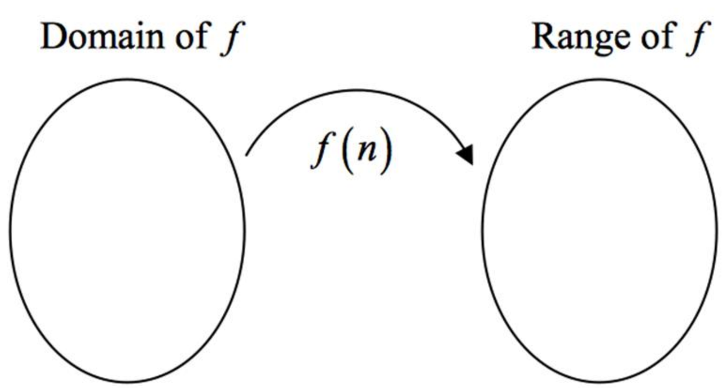
Domain: the set of all x-values.
Range: the set of all y-values.

Example: Consider the relation that has as its inputs the months of the year and as its outputs the number of days in each month. In this case, the number of days is a function of the month of the year. Assume this function is restricted to non-leap years.

Write the set that represents this function's domain:

Write the set that represents this function's range:

Example: State the range of the function $f(n) = 2n + 1$ if its domain is the set {1, 3, 5}. Show the domain and range in the mapping diagram below.



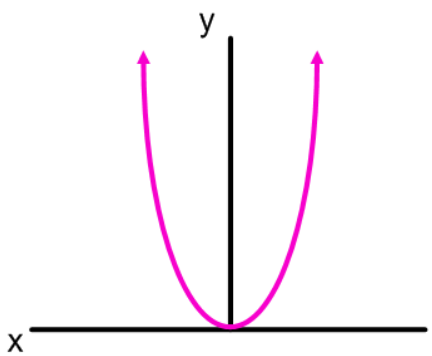
A **function** is a specific type of relation. In order for a relation to be a function there must be only and exactly one y that corresponds to a given x. This means that elements of the domain (x) never repeat. "If I am x, I only belong to one y."

Function Tests:

Graphs: Vertical line test

Note: this can also be helpful with equations – use your calculator to see a graph!

1.)



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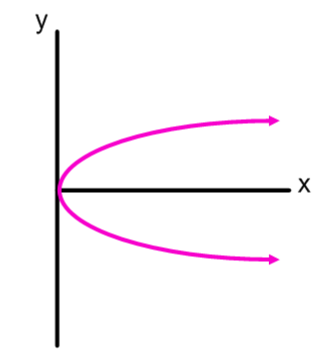
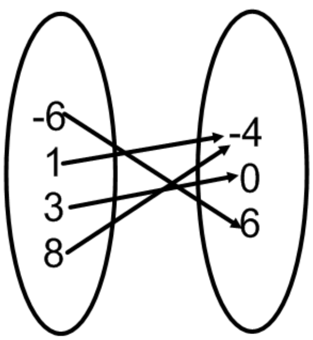


Mappings: If it is a function, each member of the __________ can only have one line drawn ________________

3.)



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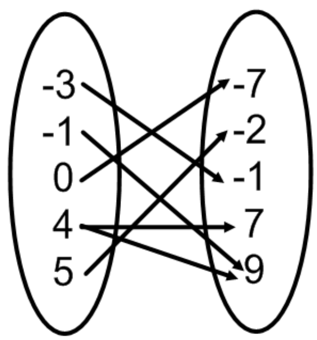


Equations: If ____________________________ in any way, it cannot be a function

Try graphing if possible!

5.) Is the relation $x^2 + (y - 1)^2 = 4$ a function? Why or why not?

HINT: what type of shape does it make?

6.) Is the relation $y = 2x + 2$ a function? Why or why not?

Evaluating Functions

Plug in to the proper function, and evaluate carefully.

Example: Given $f(x) = x^2 + 3x + 5$

(a) $f(2)$

(b) $f(x+3)$

(c) $f(-x)$

(d) $f(2\sqrt{x})$

(e) $f(x+h)$

Definition: A function that is defined by two or more equations over a specified period of time (domain) is called a piecewise function.

Example: Given the **piecewise** function: $f(x) = \begin{cases} (-2)^x, & x < -4 \\ -|x|, & -4 \le x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$

(a) $f(-4)$

(b) $f(4)$

(c) $f(-6)$

Example: $f(x) = \begin{cases} \frac{x^2-6}{x^2+6} & x \neq 0 \\ -20 & x = 0 \end{cases}$

Find:

a) $f(0)$

b) $f(-4)$

(c) $f(\sqrt{3})$

Name: ______________________________ Date: ______________ Period: ______

Basics of Functions Homework

Determine whether each is a function. State the Domain and Range.

1. $\{(1, 2), (3, 4), (5, 5)\}$

2. $\{(5, 6), (5, 7), (6, 6), (6, 7)\}$

3. $\{(-7, -7), (-5, -5), (-3, -3), (0, 0)\}$

4. $x + y = 16$

5. $y = \sqrt{x + 4}$

6. $x + y^3 = 27$

Evaluate and simplfy.

$$f(x) = \frac{4x^3 + 1}{x^3}$$

7. **a.** $f(2)$ **b.** $f(-2)$ **c.** $f(-x)$

8. $f(x) = \frac{|x + 3|}{x + 3}$

a. $f(5)$ **b.** $f(-5)$ **c.** $f(-9 - x)$

9. $f(x) = \frac{x}{|x|}$

a. $f(6)$ **b.** $f(-6)$ **c.** $f(r^2)$

Name: ______________________________ Date: ______________ Period: ________

Chapter 1: Graphs, Functions & Models
Topic 2: Restricted Domains

Restricted Domains

The domain of functions is usually unlimited ($-\infty < x < \infty$) There are two main instances where the domain of a function must be limited. Brainstorm together to find the limitations of two examples:

1. $f(x) = \frac{3x-7}{5x+15}$ Restriction:

2. $f(x) = \sqrt{x+4}$ Restriction:

3. $f(x) = \frac{x+8}{\sqrt{2x-10}}$ Restriction:

Find the domain of each of the following.

1. $y = \frac{1}{x-1}$
2. $y = \sqrt{x+5}$
3. $f(x) = \sqrt{3x+12}$
4. $g(x) = \frac{x^2+2x+1}{3x^2-12}$
5. $h(x) = \frac{x-3}{\sqrt{5x+7}}$
6. $y = 3x + 1$
7. $y = |2x + 9|$
8. $f(x) = \frac{\sqrt{5x+10}}{x-8}$

Name: ______________________________ Date: ______________ Period: ______

Restricted Domain Homework

Find the domain of each of the following functions.

1.) f(x) = $\sqrt{2x-6}$

2.) f(x) = $\frac{x^2-4}{x^2-9}$

3.). h(x) = 5x + 5

4.) g(x) = $\frac{1}{\sqrt{3x+5}}$

5.) h(x) = $\frac{x+5}{5x+11}$

7.) g(x) = x^2 + 12x + 20

8.) f(x) = $\sqrt{x^2+3x+2}$

9.) f(x) = $\frac{1}{\sqrt{x+7}}$

10.) f(x) = $\frac{11}{3x+2}$

Name: ______________________________ Date: ________________ Period: ______

Chapter 1: Graphs, Functions & Models
Topic 3: Difference Quotient

Difference Quotient

In linear equations, the slope represents the rate of change, which is constant. In other functions, the rate of change may not be constant throughout the entire function. But, the Difference Quotient provides an average rate of change for the function.

Definition of a Difference Quotient:

$$\frac{f(x+h) - f(x)}{h}$$

Where x = starting x-value
h = how far to your next x-value

Example: $x^2 + 3x + 5$

1. Evaluate $f(x + h)$

2. Plug into the full equation
and simplify

Example: $x^2 - 7x + 3$

Find and simplify the difference quotient.

1. $f(x) = x + 1$

2. $f(x) = 2x - 3$

3. $f(x) = x^2$

4. $f(x) = 5$

5. $f(x) = \frac{2}{x}$

6. $f(x) = \sqrt{x + 5}$

Name: ____ Date: ___________________ Period: _______

Difference Quotient Homework

Find and simplify the difference quotient.

$$\frac{f(x+h)-f(x)}{h},\ h \neq 0$$

1. $f(x) = 4x$

2. $f(x) = 6x + 1$

3. $f(x) = x^2 - 4x + 3$

4. $f(x) = x^2 - 5x + 8$

5. $f(x) = -x^2 + 2x + 4$

6. $f(x) = -3x^2 + 2x - 1$

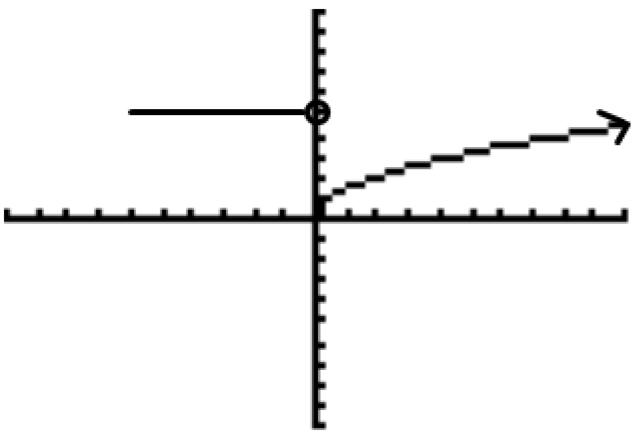
Name: ______________________________ Date: ______________ Period: ______

Chapter 1: Graphs, Functions & Models
Topic 4: Identifying Key Features of a Graph

Key Features of a Graph

A function is increasing when its graph rises,
decreasing when its graph falls, and
remains constant when its graph neither rises nor falls.

The x-values are used to state when a function is increasing, decreasing, or constant.
(use open intervals)



What is happening?

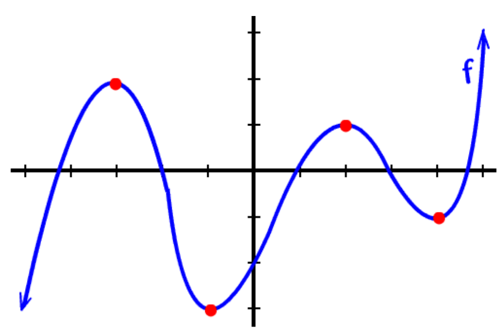
Relative maxima and Relative minima
The points at which a function changes its increasing or decreasing behavior can be used to find the relative maximum and relative minimum values of the function. There are sometimes called LOCAL minimums and maximums as well. Points including ∞ would never be considered a relative min or max

Relative maximum or relative minimum is an x-value that is greater or less than the values near it.

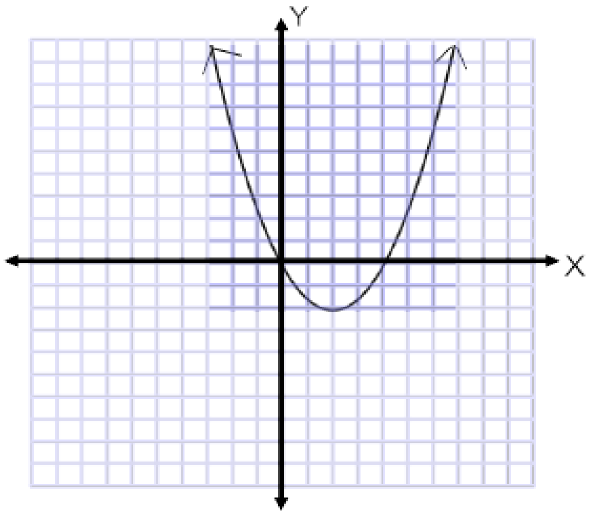
Example:
State the coordinates of:

all relative minima

all relative maxim



1.



a)What are the function values f(0), f(-1), f(2), and f(6)?

b)What are the x-intercepts (called zeros of the function)?

c)What are the y-intercepts?

d)State the intervals where the function is increasing?

e)State the intervals where the function is decreasing?

f) Are there any relative maximum or minimum?

2.

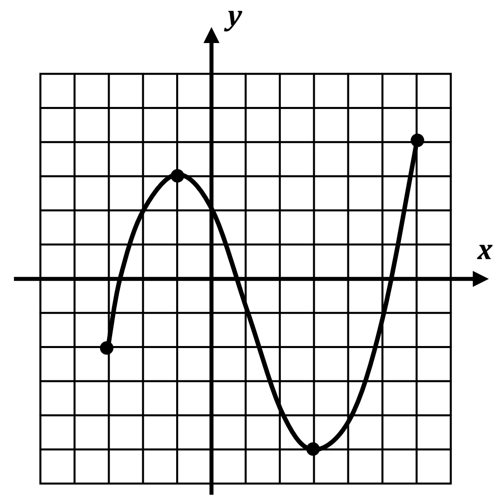
a) Evaluate:

$f(1) =$

$f(3) =$

$f(x) = -5$

For how many values does $f(x) = 3$?



b) What is the domain and range of this graph?

c) Is this a function? How do you know?

d) State an interval where the function is:

Increasing

Decreasing

Constant

e) What is the average rate of change between the points (-1,3) and (5, -1)

3.

Find:

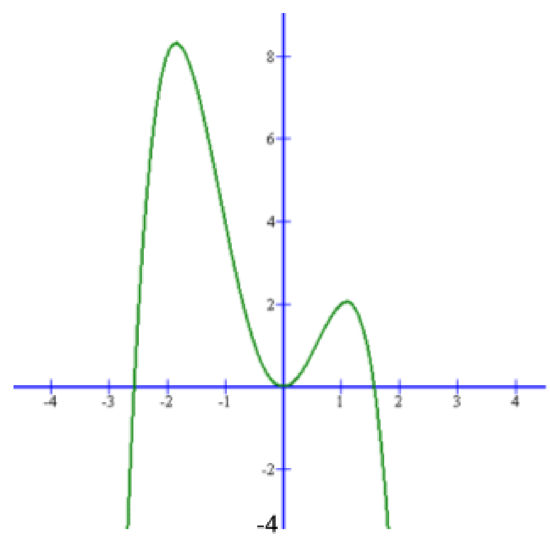
a)the x- and y-intercepts

b)intervals where f is increasing

c)intervals where f is decreasing

d)relative maximum

e)relative minimum



Real Life Examples in Graphic Form

The function $f(x) = -0.016x^2 + 0.93x + 8.5$ models the average number of paid vacation day workers receive each year nationwide, as shown in the graph.

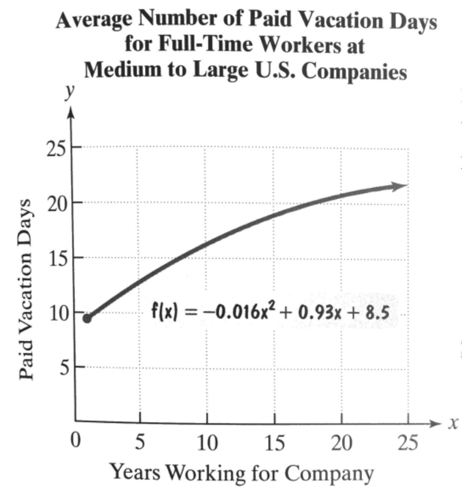


Figure 1.40
Source: Bureau of Labor Statistics

a) **About** how many vacation days does an employee receive after 5 years?

b) After **roughly** how many years will an employee be receiving 20 days of vacation per year?

c) Describe the general trend shown by the graph.

Odd & Even Functions (or neither!)

Definitions:

A function is EVEN if $f(-x) = f(x)$

The equation does not change if x is replaced with $-x$

A function is ODD if $f(-x) = -f(x)$

Every term in the equation changes sign when replaced with $-x$

If neither of these statements is true, then the function is neither odd nor even.

Examples:

Evaluate $f(-x)$

- **If it matches exactly: EVEN**
- **If it doesn't: Test ODD?**
  Maybe Neither?

$f(x) = x^4 - 2x^2$

$f(x) = x^3$

$f(x) = x^2 + 2x + 1$

Odd & Even Functions and their symmetries

Let's look at the graphs of each of the above examples

$f(x) = x^4 - 2x^2$

Notice that this function is symmetrical about the ___________________

$f(x) = x^3$

Notice that this function is symmetrical about the _______________________

$f(x) = x^2 + 2x + 1$

Notice that this function is only symmetrical about its own _______________________. *Since that symmetry line is unique to this function, it is not a general rule for 'neither' and symmetry.*

Summary:

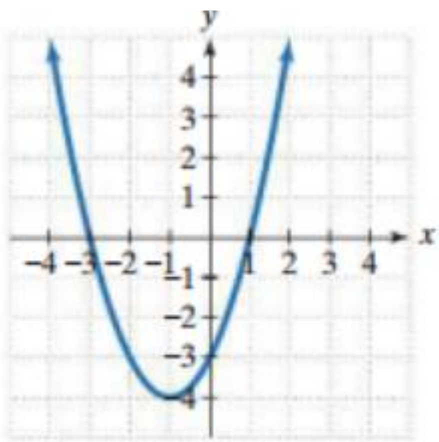
Identify each as even, odd, or neither. Then, identify any known symmetries.

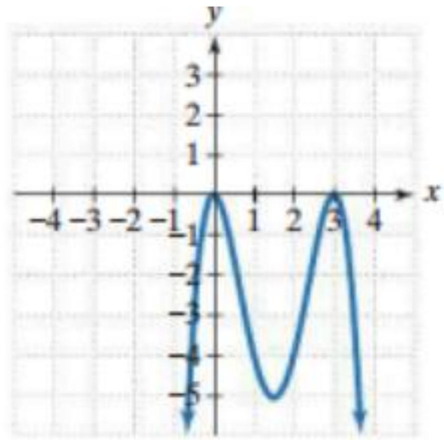
1. $g(x) = x^4 - 2x^2$
2. $h(x) = x^2 + 2x + 1$
3. $j(x) = x^2 + 6$
4. $k(x) = 7x^3 - x$
5. $f(x) = x^5 + 1$
6. $h(x) = x\sqrt{1 - x^2}$

Name: ______________________________ Date: ____________ Period: ____

Features of a Graph Homework

Identify intervals where the graph is increasing, decreasing or constant.

1. 

2. 

Determine any symmetry in the following graphs. (x-axis, y-axis, origin)

3. $y = x^2 + 6$

4. $y^2 = x^2 - 2$

5. $y = 2x + 3$

6. $x^2 + y^2 = 49$

7. $x^2 - y^3 = 2$

Determine if the function is even, odd, or neither.

8. $f(x) = x^3 + x$

9. $h(x) = 2x^2 + x^4$

10. $f(x) = x^2 - x^4 + 1$

11. $g(x) = x^2 + x$

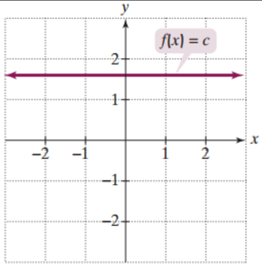
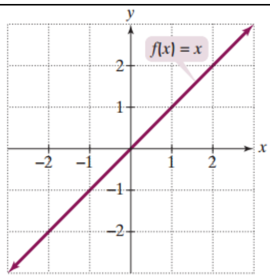
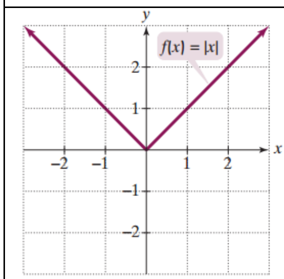
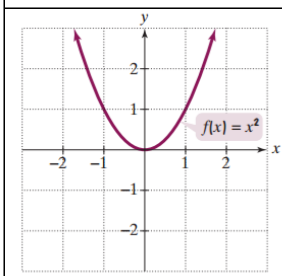
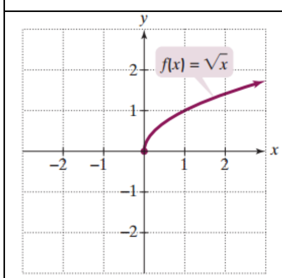
12. $f(x) = x\sqrt{1 - x^2}$

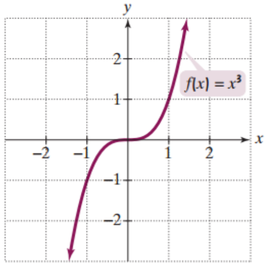
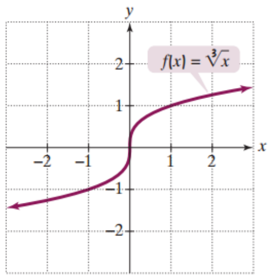
13. $f(x) = x^3 - x$

Name: ______________________________ Date: ________________ Period: ________

Chapter 1: Graphs, Functions & Models
Topic 5: Shifts in Graphs

Algebra's Common Graphs

	Name	Domain & Range	Increase & Decrease	Even or Odd
		Domain: Range:		
		Domain: Range:		
	Parent Function:	Domain: Range:		
	Parent Function:	Domain: Range:		
	Parent Function:	Domain: Range:		

	Parent Function:	Domain: Range:		
	Parent Function:	Domain: Range:		

Horizontal Shifts:

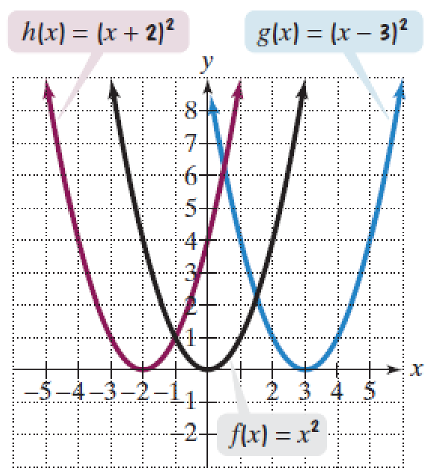
Horizontal Shifts move graphs **left & right** along the horizontal x-axis. Since the effect is happening to the x-values, the amount of the shift can be found grouped in with the x-piece of the function.

Parent:	$f(x) = \|x\|$	$f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = x^3$	$f(x) = \sqrt[3]{x}$
Shifted:	$f(x) = \|x - c\|$	$f(x) = (x - c)^2$	$f(x) = \sqrt{x - c}$	$f(x) = (x - c)^3$	$f(x) = \sqrt[3]{x - c}$

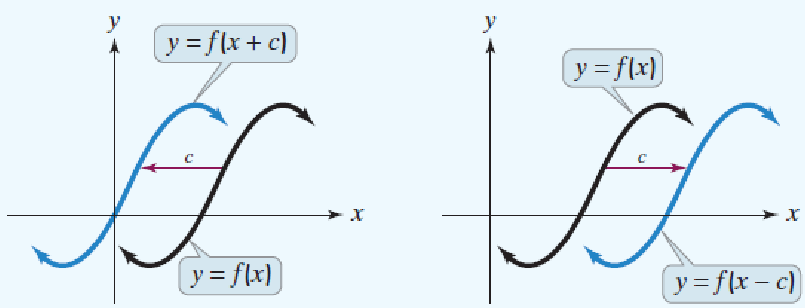
From the parent function $f(x) = x^2$, observe the graph of two shifts

- $g(x) = (x - 3)^2$ moved three units to the **RIGHT**
- $h(x) = (x + 2)^2$ moved two units to the **LEFT**

*note: these should feel 'backwards' because of the subtraction in the default shift form.



The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.
The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.



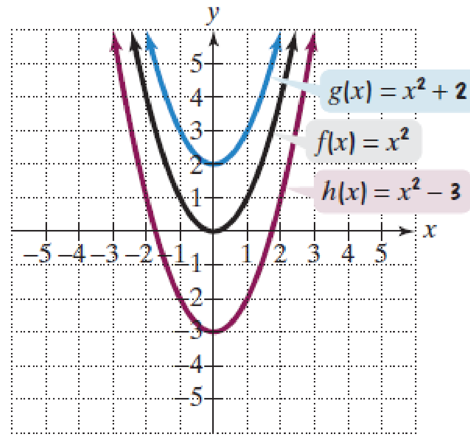
Vertical Shifts:

Vertical Shifts move graphs **up & down** along the vertical y-axis. Since the effect is happening to the y-values, the amount of the shift can be found outside of the x-piece of the function

Parent:	$f(x) = \lvert x \rvert$	$f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = x^3$	$f(x) = \sqrt[3]{x}$
Shifted:	$f(x) = \lvert x \rvert + c$	$f(x) = (x)^2 + c$	$f(x) = \sqrt{x} + c$	$f(x) = (x)^3 + c$	$f(x) = \sqrt[3]{x} + c$

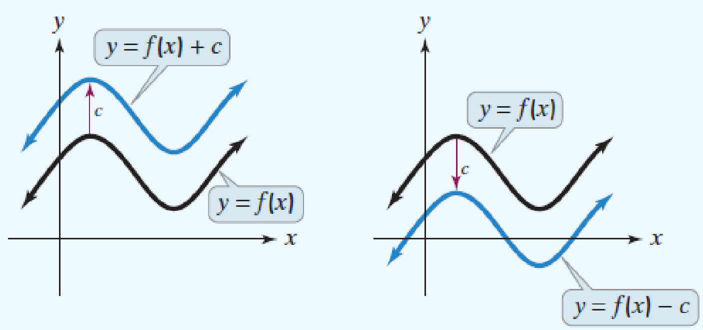
From the parent function $f(x) = x^2$, observe the graph of two shifts

- $g(x) = x^2 + 2$ moved two units **UP**
- $h(x) = x^2 - 3$ moved three units **DOWN**



The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upward.

The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downward.



Describe the shift.

1. $f(x) = |x + 3| - 2$

2. $y = (x - 2)^3 + 7$

Reflections of Graphs:

A negation in the function can cause the graph to reflect in an axis. Which axis depends on which part of the function is negated.

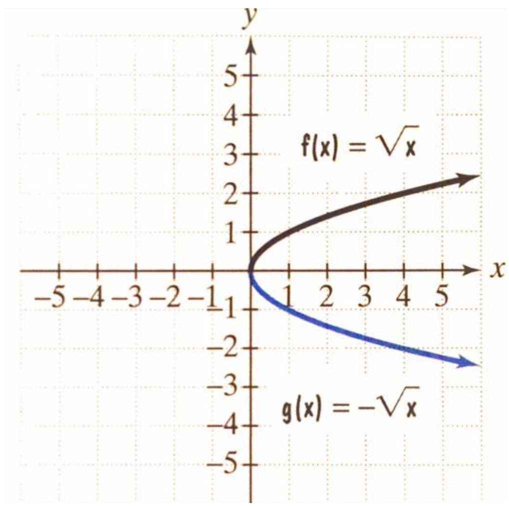
Parent:	$f(x) = \|x\|$	$f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = x^3$	$f(x) = \sqrt[3]{x}$
Reflect in X-AXIS:	$f(x) = -\|x\|$	$f(x) = -(x)^2$	$f(x) = -\sqrt{x}$	$f(x) = -(x)^3$	$f(x) = -\sqrt[3]{x}$
Reflect in Y-AXIS:	$f(x) = \|-x\|$	$f(x) = (-x)^2$	$f(x) = \sqrt{-x}$	$f(x) = (-x)^3$	$f(x) = \sqrt[3]{-x}$

Compare two types of reflection from the parent graph $f(x) = \sqrt{x}$

Reflection in the x-axis

The entire radical is negated

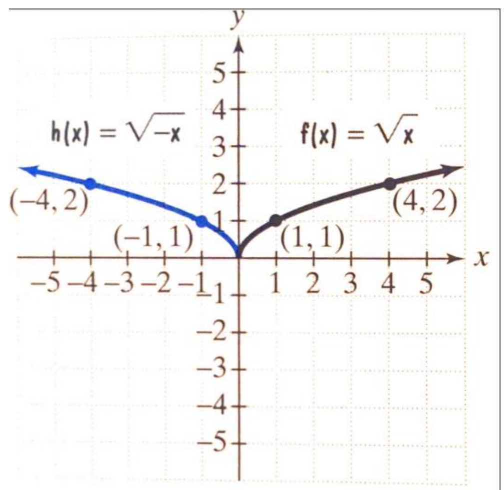
$g(x) = -\sqrt{x}$



Reflection in the y-axis

Just the x is negated

$h(x) = \sqrt{-x}$



The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

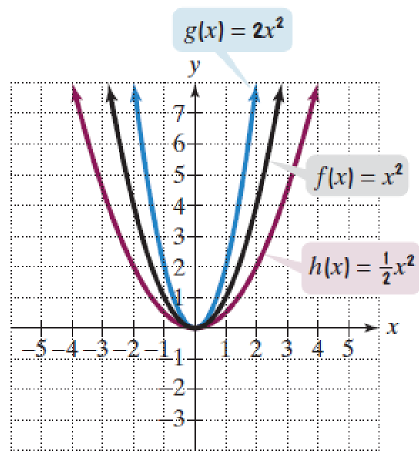
The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

Stretching & Shrinking:

Making a graph wider or narrower is effected by the coefficient of the parent term.

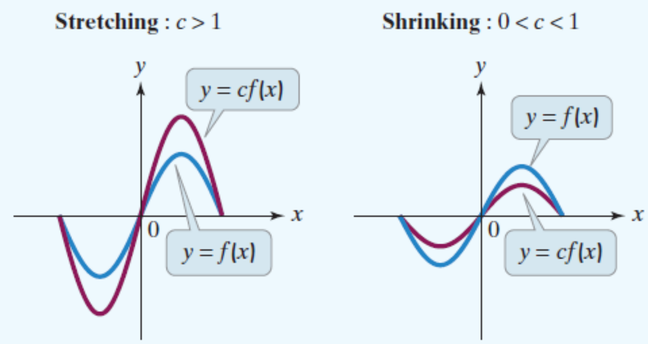
- If the coefficient is **greater than 1**, the **faster** the graph is moving upward; the **narrower** the graph
- If the coefficient is **between 0 and 1**, the **slower** the graph is moving upward; the **wider** the graph

Consider the parent function $f(x) = x^2$
$g(x) = \mathbf{2}x^2$ is **NARROWER**
$h(x) = \frac{\mathbf{1}}{\mathbf{2}}x^2$ is **WIDER**



If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of its y-coordinates by c.

If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of its y-coordinates by c.



An important note on flipping and stretching!

The coefficient of the parent term should be handled in two parts:

- A negative in the coefficient causes the flip
- A number in the coefficient causes stretching or shrinking.

Example: $f(x) = -4(x)^3$ would have:

- A reflection over the x-axis – because of the negative
- A more narrow appearance – because of the 4 (greater than 1)

Combined Examples: Describe all of the shifts from each parent graph. Use your graphing calculator to verify.

1. $y = (x+5)^2 + 9$

2. $f(x) = -\sqrt{x-6}$

3. $y = 8|x| - 7$

4. $f(x) = 5(x-4)^2 + 7$

5. $f(x) = -\frac{1}{2}|x+2| - 11$

6. $f(x) = 0.8\sqrt{x} + 18$

7. $y = 2(-x)^2 + 3$

8. $f(x) = -3\sqrt{x+9} - 2$

9. $y = \frac{2}{3}|-x| - 1$

Summary (Given that c represents a positive real number)

To Graph:	Draw the Graph of f and:	Changes in the Equation of $y = f(x)$
Vertical shifts $y = f(x) + c$	Raise the graph of f by c units.	c is added to $f(x)$.
$y = f(x) - c$	Lower the graph of f by c units.	c is subtracted from $f(x)$.
Horizontal shifts $y = f(x + c)$	Shift the graph of f to the left c units.	x is replaced with $x + c$.
$y = f(x - c)$	Shift the graph of f to the right c units.	x is replaced with $x - c$.
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f about the x-axis.	$f(x)$ is multiplied by -1.
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f about the y-axis.	x is replaced with $-x$.
Vertical stretching or shrinking $y = cf(x), c > 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically stretching the graph of f.	$f(x)$ is multiplied by c, $c > 1$.
$y = cf(x), 0 < c < 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically shrinking the graph of f.	$f(x)$ is multiplied by c, $0 < c < 1$.

Name: ______________________________ Date: ______________ Period: ______

Shifts in Graphs Homework

For each of the following, DESCRIBE the shift g(x) places on the parent function f(x).

1. $g(x) = f(x) + 1$
2. $g(x) = f(x) - 1$
3. $g(x) = f(x + 1)$
4. $g(x) = f(x - 1)$
5. $g(x) = f(x - 1) - 2$
6. $g(x) = f(x + 1) + 2$
7. $g(x) = f(-x)$
8. $g(x) = -f(x)$
9. $g(x) = -f(x) + 3$
10. $g(x) = f(-x) + 3$
11. $g(x) = \frac{1}{2}f(x)$
12. $g(x) = 2f(x)$
13. $g(x) = f\left(\frac{1}{2}x\right)$
14. $g(x) = f(2x)$
15. $g(x) = -f\left(\frac{1}{2}x\right) + 1$
16. $g(x) = -f(2x) - 1$

Name: ______________________________ Date: ______________ Period: ________

Chapter 1: Graphs, Functions & Models
Topic 6: Combinations & Composite Functions

Recall Function Notation:

Compare:

Equation: $y = 2x + 1$

Function Notation: $f(x) = 2x + 1$
Read out-loud as "f of x equals 2x+1"

Examples with inputs:

1.) Given $f(x) = x^2 + 2$, $g(x) = x^2 - x$, and $h(x) = 0.3x^2 - 3x - 2.7$ find each value:

(a) $f(-3)$

(b) $g\left(\frac{1}{3}\right)$

(c) $h(2x)$

Examples with outputs:

2.) Given $f(x) = 4 - \frac{1}{2}x$, find x such that $f(x) = 1$

3.) If $f(x) = 2x - 2$ and $f(x) = 10$, what is the value of x?

4.) Given $g(x) = x^2 - 3x - 1$, find x such that $g(x) = 3$.

Composition of Functions

In composition of functions, also known as composite functions, we perform two operations, one following the other. The answer to the first function determines our second.

Notation:
There are two ways that composition of functions are written:

(1) $f(g(x))$ (2) $(f \circ g)(x)$ ∘ means "following"

Read as: "f of g of x" "f following g of x"

Start with the function that is **closest** to whatever is in the parenthesis.
The process that you use to evaluate the functions is the same!

<u>Evaluating composite functions, examples:</u>

1) If f(x) is defined as $f(x) = x + 2$ and g(x) is defined as $g(x) = x^2 - 4$, find $f(g(3))$.

Start from the inside, underlining the number and closest function

Evaluate that expression

Draw an arrow placing the result in the outer function

Evaluate that expression

Write your answer

2) If f(x) is defined as $f(x) = 2x + 3$ and h(x) is defined as $h(x) = x^2 + 1$, find:

a) $(h \circ f)(-5)$ b) $f(h(-4))$

3) If $f(x) = x^2 - 2x + 1$ and $g(x) = 3x - 4$, find:

a) $g(f(-2))$ b) $f(g(-2))$

4) If f(x) is defined as f(x) = x + 2 and g(x) is defined as g(x) = x^2 - 4, find:

a) $f(g(2x))$ b) $g(f(x+2))$ c) $g(f(x))$

Operations on Functions

Split & evaluate separately, then combine by the given operation.

Operation	Notation
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Examples:

If $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$, find

(a) $(f+g)(7)$

(b) $(f-g)(-2)$

(c) $\left(\frac{f}{g}\right)(3)$

(d) $f(g(4))$

Name: ______________________________ Date: ______________ Period: ________

Combinations and Compositions Homework

For each set of functions, find:

a. $(f \circ g)(x)$ ***b.*** $(g \circ f)(x)$

1. $f(x) = 2x, g(x) = x + 7$

2. $f(x) = 5x + 2, g(x) = 3x - 4$

3. $f(x) = 4 - x, g(x) = 2x^2 + x + 5$

4. $f(x) = \sqrt{x}, g(x) = x + 2$

5. $f(x) = 2x - 3, g(x) = \dfrac{x + 3}{2}$

6. $f(x) = x^2 + 1, g(x) = x^2 - 3$

Name: ______________________________ Date: ______________ Period: ________

Chapter 1: Graphs, Functions & Models
Topic 7: Inverse Functions

The **inverse of a function** is a relation in which the domain and range of the original function have been exchanged (x and y have switched places)

Symbol: $f^{-1}(x)$ read as "f inverse of x"

Note: Even if the inverse itself does not form a function, this notation can still be used.

Ordered Pairs, Tables & Mappings

The inverse of a function is formed by interchanging the x and y coordinates of each pair in the function.

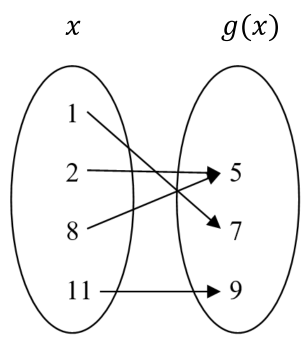
Examples: Find the inverse of each given function below

1. Given f(x) = {(0,1), (9,2), (8,3), (7,4)} find $f^{-1}(x)$.

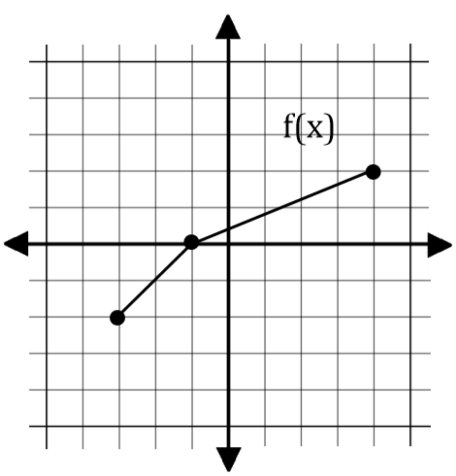
2.

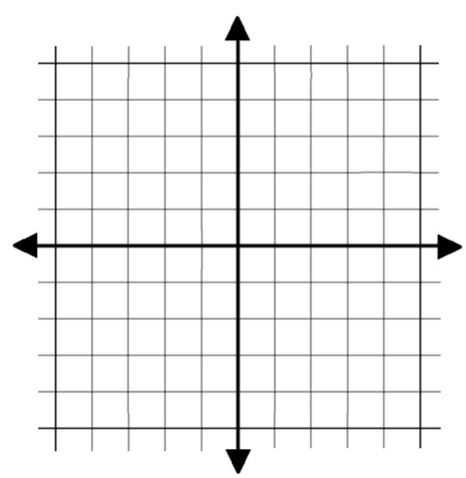
x	$h(x)$
-10	9
7	-11
11	1
23	0

3.



3. (write a table of values, flip, graph)





Equations

The process is still mainly switching the x & y values. But, in equation form, we need to re-solve for y to have the function in proper form.

Example: Find the inverse of $f(x)$ if $f(x) = x - 8$

a) Express the function as y

b) Switch x and y to form the inverse

c) Solve for y

d) Express in inverse notation

Examples:

1. $f(x) = 3x + 5$

2. $g(x) = \frac{1}{2}x - 6$

3. $f(x) = \frac{x+6}{3}$

4. $y = \sqrt{x - 2}$

5. $g(x) = 3x^3 - 6$

6. $y = (3x + 2)^3$

7. $f(x) = \frac{5}{x}$

8. $y = x^2 + 5$

9. $g(x) = \sqrt[3]{x} - 6$

Computing with Inverse Functions

Inverse functions serve to "undo" one another.

For example: Find the inverse function of $f(x) = \frac{2x+7}{3}$

Compute:

(a) $f\left(f^{-1}(5)\right)$

(b) $f^{-1}\left(f(4)\right)$

(c) $f\left(f^{-1}(x+1)\right)$

(d) Predict: $f^{-1}\left(f(x^3 - 2x^2 + 7)\right)$

Name: ______________________________ Date: ______________ Period: ______

Inverse Functions Homework

Find f(g(x)) and g(f(x)) and determine whether each pair of functions are inverses of one another.

1. $f(x) = 4x$ and $g(x) = \frac{x}{4}$

2. $f(x) = 3x + 8$ and $g(x) = \frac{x - 8}{3}$

3. $f(x) = \sqrt[3]{x - 4}$ and $g(x) = x^3 + 4$

4. $f(x) = 4x + 9$ and $g(x) = \frac{x - 9}{4}$

Find the inverse of the following functions.

5. $f(x) = x + 3$

6. $f(x) = 2x + 3$

7. $f(x) = x^3 + 2$

8. $f(x) = (x + 2)^3$

9. $f(x) = \dfrac{2x + 1}{x - 3}$

10. $f(x) = \dfrac{2x - 3}{x + 1}$

Chapter 1: Graphs, Functions & Models
Topic 8: Circles

Standard Equation of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

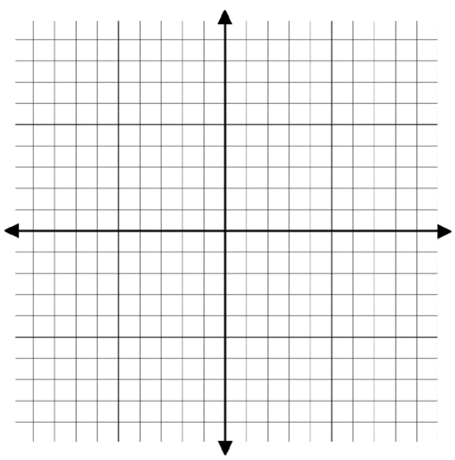
where (h, k) is the center and r is the radius

A circle can be represented in three ways:
- Equation
- Center & Radius
- Graphed

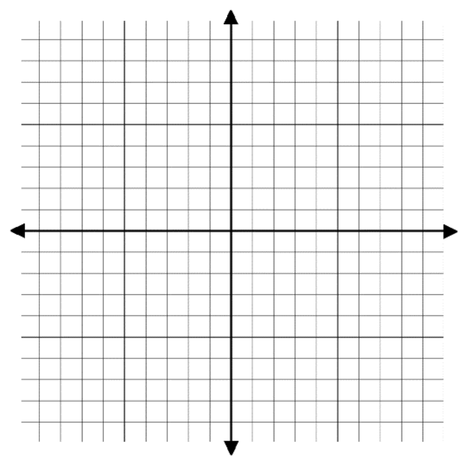
If we know one of these, we can determine all three.

Examples: Represent the circle in the other two ways:

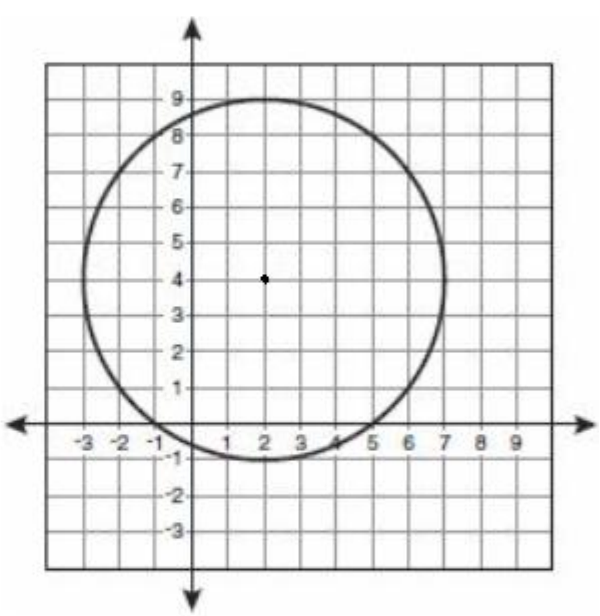
1. A circle with a center at (5, -3) and a radius of 4



2. $(x+4)^2 + (y+1)^2 = 36$



3.



General Equation of a Circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

To convert from general form to standard form:

$x^2 + y^2 + 4x - 6y - 23 = 0$

Rearrange: Put the x's together and the y's together.
Move the constant.

Complete the square: Twice. Once for x's, once for y's.

Clean up.

Examples: Rewrite the equation in standard form and identify the center and radius.

1. $x^2 + y^2 - 4x + 2y - 4 = 0$

2. $x^2 + y^2 - 10x - 6y - 30 = 0$

3. $x^2 + y^2 + 2x + 6y + 1 = 0$

4. $x^2 + y^2 - 4x - 6y - 3 = 0$

Name: ______________________________ Date: ________________ Period: ______

Circles Homework

For each of the following sets of points, find:

a. the distance between the two points rounded to the nearest hundredth

b. the midpoint between the two points

1. $(2, 3)$ and $(14, 8)$

2. $(2, -3)$ and $(-1, 5)$

3. $(-2, -6)$ and $(3, -4)$

4. $(3.5, 8.2)$ and $(-0.5, 6.2)$

Give the center and radius of the following circles.

5. $x^2 + y^2 = 16$

6. $(x - 3)^2 + (y - 1)^2 = 36$

7. $x^2 + (y - 2)^2 = 4$

8. $(x + 1)^2 + y^2 = 25$

Put each of the following circles in standard form.

9. Center $(2, -1)$, $r = 4$

10. Center $(-5, -3)$, $r = \sqrt{5}$

11. $x^2 + y^2 + 6x + 2y + 6 = 0$

12. $x^2 + y^2 + 8x + 4y + 16 = 0$

13. $x^2 + y^2 - 10x - 6y - 30 = 0$

14. $x^2 + y^2 - 4x - 12y - 9 = 0$

15. $x^2 + y^2 + 8x - 2y - 8 = 0$

16. $x^2 - 2x + y^2 - 15 = 0$

17. $x^2 + y^2 - x + 2y + 1 = 0$

18. $x^2 + y^2 + 12x - 6y - 4 = 0$